

Portfolio Optimization

This approach to portfolio management combines the ability to effectively identify the optimal frontier with a novel interactive graphical decision-making method that allows the user to quickly down-select to a small subset of desirable portfolios.

Investors know that there is a risk-return tradeoff. In order to obtain greater returns on investments, the investor must be willing to take on greater risk. Portfolio theory assumes that for a given level of risk, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer lower risk to greater risk. In addition, one can assume that investors would like to invest in an efficient portfolio, that is, a portfolio in which there is no other portfolio that offers a greater return with the same or lower risk.

Mean or expected return is commonly employed as a measure of return, and variance or standard deviation of return is commonly employed as a measure of risk. In practice, mean-variance efficient portfolios have been found to be quite unstable. Typically, input parameters like expected returns, correlation among projects, and project variance are estimated using either historical data or forecasts. Researchers have found that estimation errors in the input parameters overwhelm the theoretical benefits of the mean-variance paradigm.

Over the last several years, alternative methods have emerged for optimizing uncertain financial decisions. A new portfolio management system that integrates multi-objective optimization and interactive Pareto frontier decision-making techniques is being used to improve financial asset allocation by introducing more flexible, robust, and realistic assumptions and providing more sophisticated portfolio analysis.

The Portfolio Optimization Challenge

Modern Portfolio Theory (MPT) treats portfolio optimization as a two-dimensional optimization problem, estimating risk as the variance of expected return. However, there are actually several kinds of risks, including specific risks associated with a single asset, or narrow asset type, and risks associated with changing economic conditions, political events, or other factors, which will typically affect different assets in different ways. Moreover, other ways of estimating risks often make sense. An example is the Value at

Risk model, which estimates the losses that can be expected if unusually adverse events occur. The bottom line is that investing is complex and investors can do better. They can actually achieve higher rates of return, while better controlling risks, by treating portfolio optimization as a multi-objective optimization problem with more than two dimensions. Both theoretical reasoning and some limited empirical evidence support the claim of improved performance using this approach.

Finding a Solution

Portfolio analysis tools are designed to aid senior management in the development and analysis of portfolio strategies by giving them the capability to assess the impact of various investment decisions. Advances in artificial intelligence and methods based on analogs to physical, biological, or evolutionary processes have led to the creation of optimization engines that successfully guide a series of complex evaluations with the goal of finding optimal values for decision variables.

Most real-world optimization problems have several, often conflicting objectives. Portfolio optimization is no different. Therefore, the result is not a single solution, but a set of solutions that trade-off between objectives. The Italian economist Vilfredo Pareto first formulated this concept in 1896, and it bears his name today. A solution is Pareto optimal if (for a maximization problem) no increase in any criterion can be made without a simultaneous decrease in any other criterion.¹

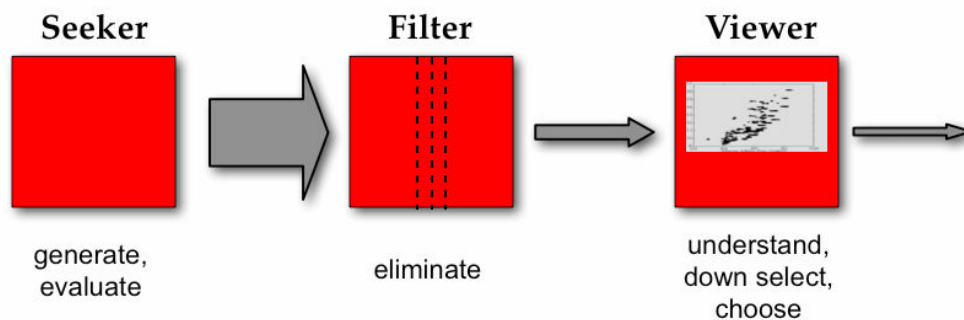
While return measures are typically linear, risk measures are typically nonlinear and often nonconvex. In a portfolio design problem with strictly linear objectives and constraints, a linear programming solution approach is the best fit. However, if one or more of the objectives are nonlinear, alternative approaches are required. For multi-dimensional portfolio design problems with linear constraints where the return measure is linear, and the risk measure is nonlinear, a portfolio optimization system is needed that integrates evolutionary multi-objective optimization and interactive Pareto frontier decision-making techniques to optimally allocate financial assets while considering multiple measures of return and risk and numerous regulatory constraints. The multi-objective optimization approach uses evolutionary computation to simultaneously maximize these return measures, minimize these risk measures, and identify the optimal frontier of portfolios that satisfy all constraints. The method combines a novel interactive

graphical decision-making method that allows the decision-maker to quickly down-select to a small subset of optimal portfolios. This approach has been tested on real-world portfolios with hundreds to thousands of assets, and is currently being used for investment decision-making.²

A Solution for Multi-Objective Portfolio Optimization

Aetion is the leader in providing patented technology that integrates multi-objective optimization and interactive, Pareto-frontier decision making. The solution, Seeker-Filter-Viewer (SFV), offers a novel multi-objective portfolio optimization algorithm that uses evolutionary computation for portfolio design problems with multiple measures of risk and return, where the measures may be nonlinear and nonconvex. In addition, an interactive graphical decision-making method allows a user to quickly down-select to a small subset of optimal portfolios via iterative constrained selections of portfolios represented graphically in several ways such as points projected in two-dimensional graphs over the combinations of the various return and risk measures utilized.³

Aetion’s offering is comprised of three functional components, the Seeker, the Filter, and the Viewer, as shown in the following figure. The “Seeker” generates alternatives by rule-governed composition of components and evaluates them according to multiple criteria. Then, the “Filter” removes the dominated alternatives, and the efficient frontier is viewed interactively in the “Viewer” by means of cross-linked diagrams wherein the same alternatives are identically colored in each diagram, enabling the comparison of alternatives from multiple perspectives.⁴

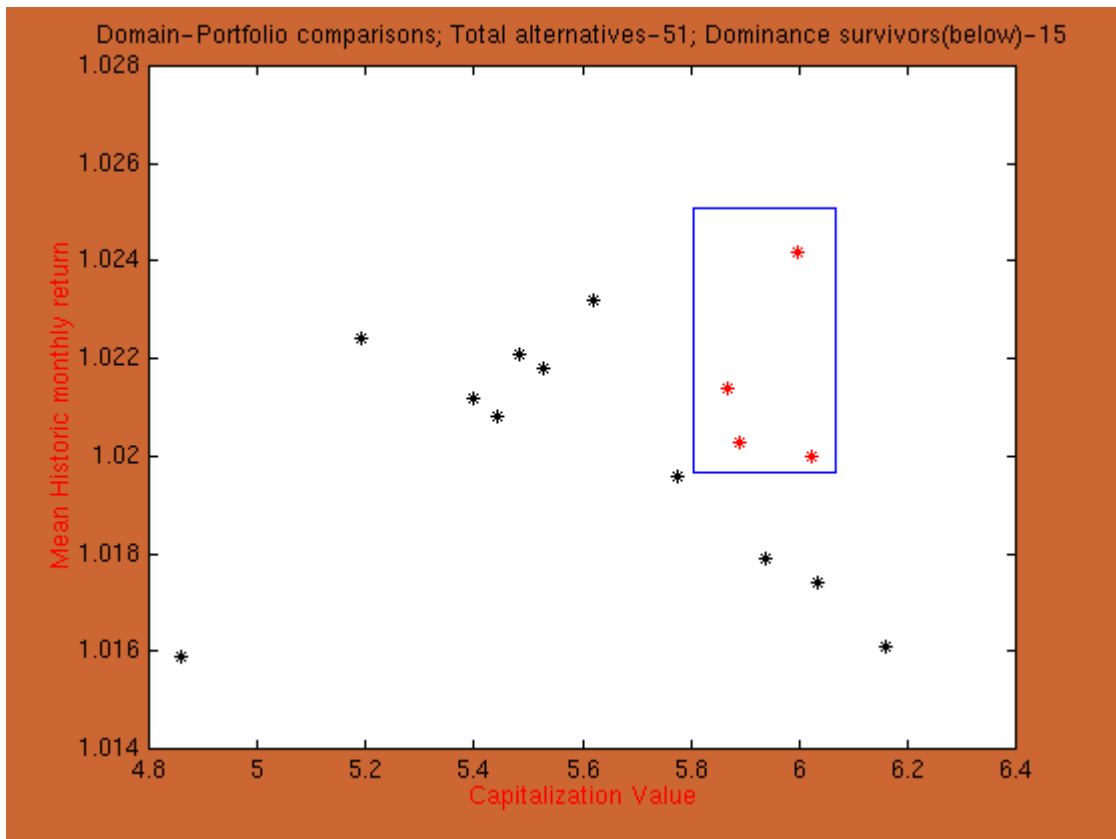


The Seeker – The Seeker generates alternatives and evaluates them according to multiple criteria using simulations and/or other computational methods. The Seeker can use any number of networked computers to evaluate portfolios in parallel. The number of portfolios evaluated is limited mainly by the number of computers available, the time available, and the time it takes to evaluate. If needed, the Seeker can use simulated evolution to generate alternatives by mutation and breeding. By considering millions of alternatives through this simulated evolution, users can effectively search among *billions of alternatives*.

Filter - The total number of alternatives might be quite large, so it is desirable to use the computer to select a relatively smaller number of alternatives that are worth further examination. The Filter can employ a method based on a dominance rule so that filtering is lossless, i.e., so that there are guarantees that no good alternatives are excluded. In other words, an alternative *A dominates* another alternative *B* if *A* is superior or equal to *B* with respect to every criterion of evaluation and distinctly superior with respect to at least one criterion. The alternatives generated by the seeker are systematically compared, and any candidate that is dominated by any other candidate is removed. The survivors comprise the *Pareto subset*. No member of the Pareto subset dominates any other element in the subset. Thus, deciding among alternatives in the Pareto subset is always a matter of tradeoffs.

Viewer – The Viewer enables decision-makers to perform interactive visual trade-off analyses of the elements of the Pareto set and to narrow to a subset for further exploration. The Viewer is used to explore the surviving design alternatives, and to visualize the trade-offs. The Viewer enables the planner to visualize performance characteristics and other attributes of sets of alternatives using various types of interlinked displays, including *trade-off diagrams*, which are two-dimensional scatter plots with points that represent alternatives and axes that represent criteria. The user can graphically select a subset of the alternatives and explore their ranges of values in multiple dimensions. In this way, the users gain an understanding of the decision space and can act to successively narrow the focus to reduce the number of alternatives under consideration. The user is able to zoom in on subsets with desirable tradeoff characteristics, and reduce the number of alternatives toward making a final selection. The Viewer is graphical, mouse driven, and browser-embeddable so it can be easily

deployed over the Internet or over an intranet. An example of a trade-off diagram appears below.



Advantages of the SFV Solution

Evidence suggests that people are poor at multiple-criteria decision making, even when there are relatively few alternatives, often picking alternatives that are clearly sub-optimal. The SFV solution is effective even when the number of alternatives is small. However, the technology works spectacularly well when the number of alternatives is very large, say, in the millions. In such situations, the technology empowers decision makers to explore many more possibilities - orders of magnitude more possibilities - than would otherwise be possible. This may be of particular importance in the area of portfolio management where even a small decision-making advantage in portfolio asset selection could mean a significant amount of money. Some additional advantages in using SFV technology in this arena includes its ability to:

- *Extend human capabilities* -- SFV technology takes advantage of human cognitive strengths, and provides computer assistance to mitigate human cognitive weaknesses. The objective is to improve human decision-making, not supplant it. This is *cognitive augmentation*, or *intelligence amplification*. The computer and the human do what each is best at: the human makes the ultimate value judgments, while the computer provides for acquiring, scoring, keeping track, and routine comparison of all the alternatives.
- *Provide insight into the decision space* - A major benefit of the approach is the *understanding* of the decision space that decision makers get as a result of exploring a vastly extended range of possibilities.
- *Manage uncertainty* -- SFV technology provides many ways in which decision makers can manage or eliminate uncertainty by considering separate measures of risk. Very large numbers of alternatives can be considered, thus reducing the chance that surprises are lurking undiscovered. The proverbial needle in the haystack can be brought to light. Parallel processing supports this large-scale exploration, so that it may be accomplished quickly if large numbers of processors are employed. The study of many evaluation results in the Viewer will highlight surprising ones that are worth further investigation.
- *Enable out of the box thinking*-- Decision makers are encouraged to consider many more possibilities than they would otherwise be able to handle. This makes discoveries outside the range of usual thinking possible where alternatives are narrowed to manage complexity.

Case Study

Background: The goal of portfolio optimization is to manage risk through diversification and obtain an optimal risk-return tradeoff. General Electric deployed a novel, evolutionary multi-objective portfolio optimization algorithm that used evolutionary computation for portfolio design problems with multiple measures of risk and return, where the measures were nonlinear and nonconvex. Participants used an interactive, graphical decision-making method to quickly down-select to a small subset of efficient portfolios via iterative constrained selections of portfolios represented as points

projected in two-dimensional graphs over the combinations of the various return and risk measures utilized.⁵

Risk Functions Used: Within this particular study, the value of a security was characterized as a function of multiple underlying risk factors. The change in the value of a security was approximated with the changes in the risk-factor values and risk sensitivities to these risk factors. The portfolio variance equation was derived analytically from the underlying value change function. The portfolios had assets and liabilities that were affected by the changes in common risk factors. Since a majority of the assets were fixed-income securities, the dominant risk factors were interest rates. In addition to maximizing return or minimizing risk, portfolio managers were constrained to match the characteristics of asset portfolios with those of the corresponding liabilities to preserve portfolio surplus due to interest rate changes. Therefore, the portfolio optimization problem formulation had additional linear constraints that match the asset-liability characteristics when compared with the traditional Markowitz model.⁶

Summary: Optimal portfolio selection is characterized by multiple objectives, measuring different types of return and risk, which need to be optimized or at least satisfied simultaneously. The decision maker (DM) needs to search for the non-dominated solutions in this objective space, while aggregating his/her preferences over multiple criteria. Since these objectives cannot be satisfied simultaneously, users need to accept tradeoffs.

This study incorporated the decision-maker's preferences in the return-risk tradeoff to perform the selection— with the goal being the reduction of thousands of non-dominated solutions into a much smaller subset, which could be further analyzed for a final portfolio selection. After obtaining a Pareto front, this space was augmented with three additional metrics to reflect additional constraints for use in the tradeoff process. This augmented space was used in the down-selection process. To incorporate progressive ordinal preferences, a graphical tool was used to visualize 2-D projections of the Pareto front—in this case the projections being (*Book-yield, Risk1*), (*Book-yield, Risk2*), (*Risk2, Risk1*), and (*Book-yield, Duration-weighted-market-Yield*).⁷

As a result of this study, the decision-makers understood the available space of options and the costs/benefits of the available tradeoffs.

Take Aways

- Investing is complex and investors can do better. They can actually achieve higher rates of return, while better controlling risks, by treating portfolio optimization as a multi-objective optimization problem with more than two dimensions. Both theoretical reasoning and some limited empirical evidence supports the claim of improved performance.
- A great advantage of the approach described here is in explicitly considering an extensive number of alternative portfolio options before estimating the Pareto front.
- Aetion is the leader in providing technology that integrates evolutionary multi-objective optimization and interactive Pareto frontier decision making. The solution, Seeker-Filter-Viewer (SFV), offers a patented, multi-objective portfolio optimization algorithm that uses evolutionary computation for portfolio design problems with multiple measures of risk and return, where the measures may be nonlinear and nonconvex. In addition, an interactive graphical decision-making method allows a decision-maker to quickly down-select to a small subset of optimal portfolios via iterative constrained selections of portfolios represented in graphs showing the combinations of the various return and risk measures utilized.⁸

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¹ R. Subbu, P. Bonissone, S. Bollapragada, K. Chalermkraivuth, N. Eklund, N. Iyer, R. Shah, F. Xue, and W. Yan, "A Review of Two Industrial Deployments of Multi-criteria Decision-making Systems at General Electric," *General Electric Global Research*, One Research Circle, Niskayuna, NY 12309, USA.

² Ibid.

³ Ibid.

⁴ M. Carroll, J. Josephson, J. Russell, Tradeoffs on the Efficient Frontier of Network Disruption Attacks, This research was prepared under sponsorship from the U.S. Naval Surface Warfare Center under contract N00178-02-C-3063, through participation in the Advanced Decision Architectures Collaborative Technology Alliance sponsored by the U.S. Army Research Laboratory under Cooperative Agreement DAAD19-01-2-0009, and by federal flow-thru by the Department of Defense under contract FA8652-03-3-0005 (as a subcontract from Wright State University and Wright Brothers Institute).

⁵ Ibid.

⁶ Ibid.

⁷ Ibid.

⁸ Ibid.